S011
A552
15

MISC. PUB. NO. 15
OCTOBER 1957

LIBRARY
MAR 8 1968
ROCKY MOUNTAIN STATION

PARALLAX WEDGE PROCEDURES IN FOREST SURVEYS

by

KARL E. MOESSNER AND EARL J. ROGERS

DIVISION OF FOREST ECONOMICS

INTERMOUNTAIN FOREST & RANGE EXPERIMENT STATION
FOREST SERVICE
U. S. DEPARTMENT OF AGRICULTURE
OGDEN, UTAH
REED W. BAILEY, DIRECTOR

Misc. Pub. No. 15 October 1957

PARALLAX WEDGE PROCEDURES IN FOREST SURVEYS

By

Karl E. Moessner and Earl J. Rogers
Division of Forest Economics

INTERMOUNTAIN FOREST AND RANGE EXPERIMENT STATION
Forest Service
U. S. Department of Agriculture
Ogden, Utah
Reed W. Bailey, Director

CONTENTS

	Page
INTRODUCTION	1
ORIENTATION AND ALIGNMENT	3
Orientation of photos	3
Alignment of wedge and stereoscope	6
MATHEMATICS OF PARALLAX MEASUREMENTS	8
Parallax at top of tree (P_t)	8
Elevation of camera above ground	10
PRACTICAL APPLICATION	12
Some tests of the methods	18
Relative cost and accuracy of measurements	19
SUMMARY	21
LITERATURE CITED	22

PARALLAX WEDGE PROCEDURES IN FOREST SURVEYS

Karl E. Moessner and Earl J. Rogers[1/]

The parallax wedge has often proved superior to much more expensive instruments for rapid estimation of tree heights from vertical aerial photographs. Since many foresters who have tried to use the parallax wedge have been only partly successful, this paper explains and demonstrates certain fundamental techniques so that a forester can make effective use of this tool in interpreting vertical aerial photographs. It brings together several important items published during the past dozen years and presents some new techniques for measurement. In connection with these are given a few demonstration exercises that will help the beginner become proficient in using the wedge.

Both Spurr (7)[2/] and Rogers (6) found the parallax wedge extremely useful on forest surveys. Moessner, working with 1:20,000-scale photos, estimated stand height in hardwoods (5) and also in conifers (4), and found that these stand heights were not significantly different from those measured on the ground. This degree of accuracy is equal to that obtained by using the most expensive stereoscopic plotting instruments.

The parallax wedge (fig. 1) consists of two converging lines, one of which is graduated to show the amount of convergence, in this case 0.002 inch. Measurements are made between lines at right angles to the long axis of the wedge. To facilitate orientation, tick marks are placed on the opposing line at the 0.05- and 0.10-inch graduations. During measurements, these ticks and their corresponding graduation must be perfectly aligned.

Various types of wedges have been designed: some consist of rows of dots instead of graduated lines; some are printed on glass, others on film. Both red- and black-lined wedges are used. Wedges having a range of 0.350 inch are typical; but for photos made in mountainous areas the long wedge (3) having a range of 1.05 inches serves better. Regardless of type or color of wedge, the principles governing its use are the same.

1/ Moessner is in the Division of Forest Economics, Intermountain Forest & Range Experiment Station at Ogden, Utah; Rogers is in the Division of Forest Economics, U. S. Forest Service, Washington, D. C.

2/ Numbers in parentheses refer to Literature Cited, page 22.

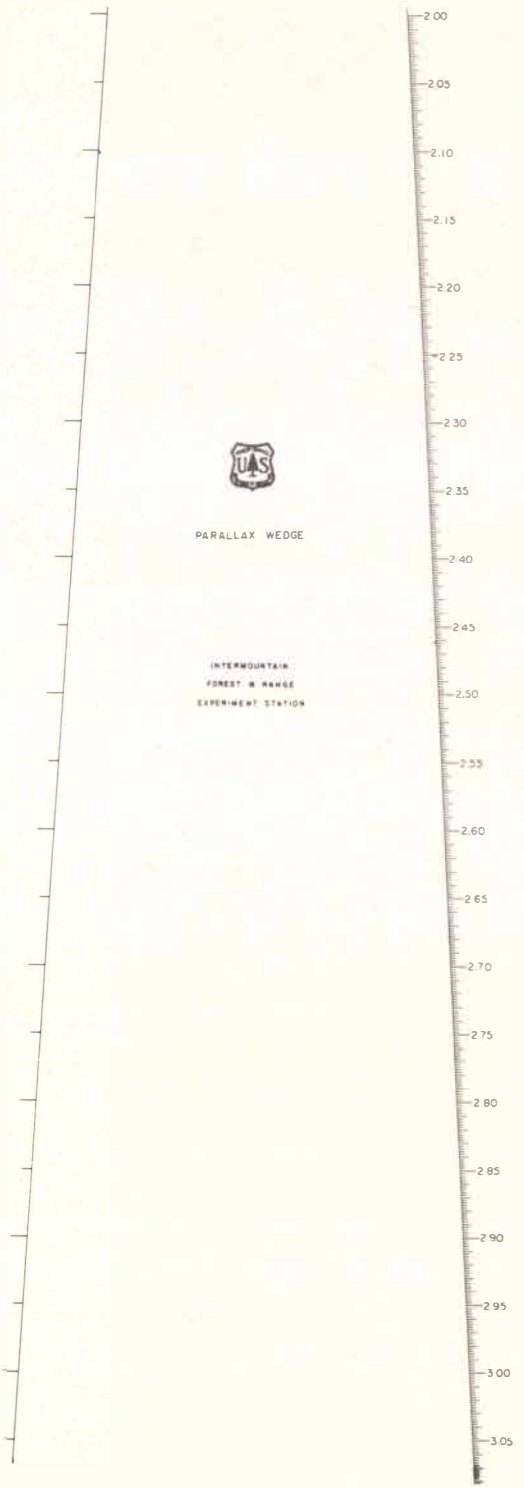

Figure 1.--The long parallax wedge allows measurements at varying elevations with one photo setup.

ORIENTATION AND ALIGNMENT

Two manual processes that necessarily precede parallax wedge measurements are orientation of the photo pair and alignment of the wedge and stereoscope with the photos. Orientation and taping down the photos is necessary since parallax differences[3/] by definition are measured parallel to the air base. Movement of the photos during measurement would result in unusable data.

Orientation of Photos

Each pair of photos must be oriented for parallax measurements with the four principal and conjugate principal points aligned. The paired photos are separated by a distance approximately equal to the midmeasurements of the wedge. Although this 2.55-inch separation allows most effective use of the parallax range of this wedge, beginning students may find a separation as small as 2.25 inches necessary during training.

A kite-shaped device (fig. 2) has been developed to aid the photo interpreter in locating principal points and in aligning his photos for parallax measurement (2). This guide is oriented over each photo by aligning its crossed lines with the four fiducial marks; then the principal point is pinpricked on the photo through the hole at the intersection of the lines.

Next, the photos are overlapped in flight order with titles on the left- and right-hand photo on top. They are separated about 2.6 inches so that the principal point of the left-hand photo may be seen stereoscopically through the lens stereoscope. The conjugate principal point is then pinpricked on the right-hand photo. With titles on the right and photo positions reversed, this operation is repeated on the left-hand photo.

The photos are again placed in line of flight with headings to the left, and the outer (left) edge of the left-hand photo is taped to the desk with an 8- or 9-inch strip of masking tape. The alignment guide is centered over the principal point of this photo with the graduated line cutting the conjugate principal point. While the guide is held firmly by the left hand, the second photo is slipped beneath it from the right until the conjugate principal point lies beneath the 2.55-inch graduation. This photo is rotated until the graduated line of the guide intersects both the principal and conjugate principal points. The second photo is then taped down along its outer (right-hand) edge. This method of taping allows

---

3/ Manual of Photogrammetry defines parallax difference as the difference in absolute stereoscopic parallaxes of two points imaged on the photos (1, p. 829). The absolute stereoscopic parallax of a point is further defined as "the algebraic difference, parallel to the air base, of the distances of the two images from their respective principal points" (1, p. 829).

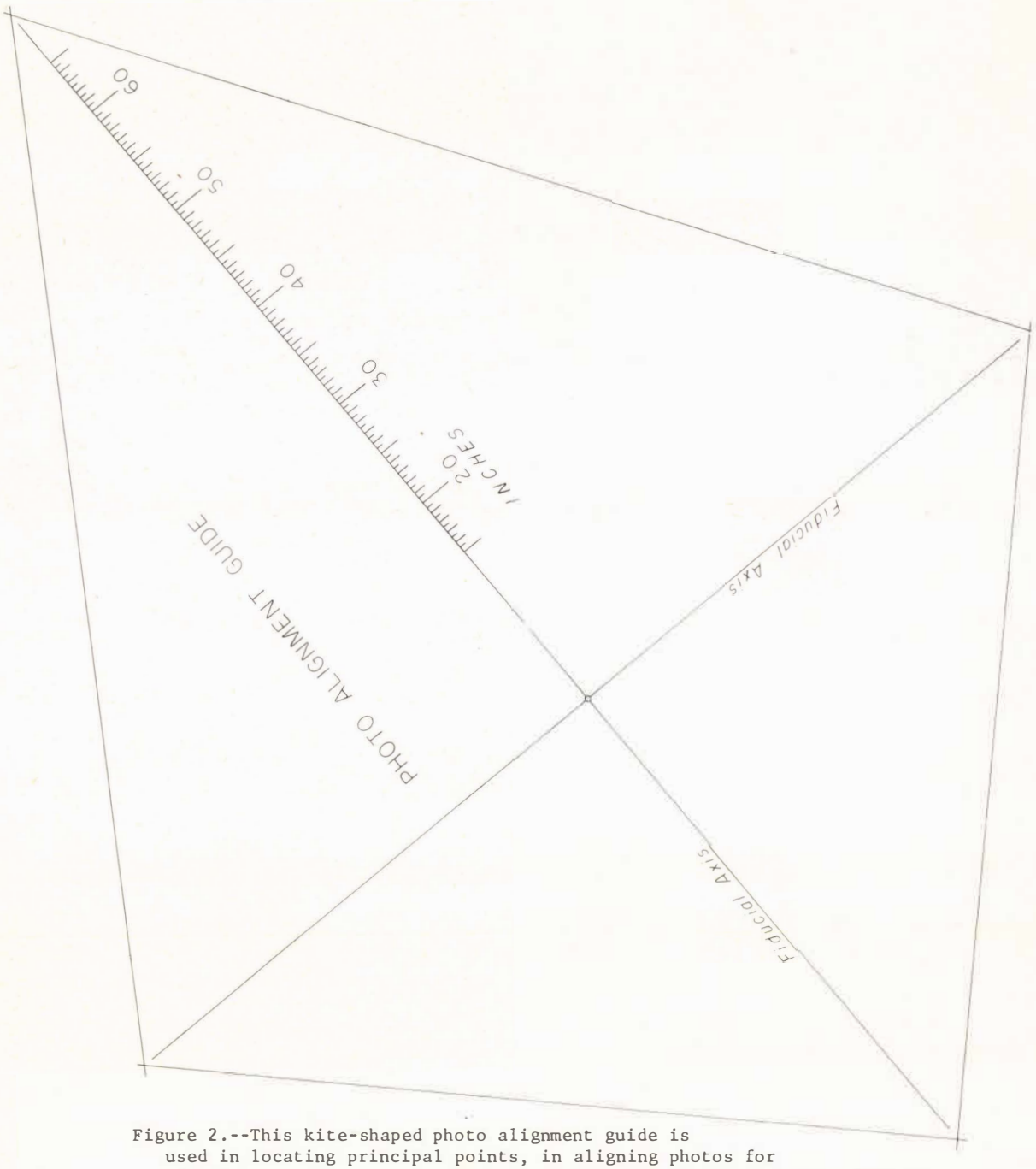

Figure 2.--This kite-shaped photo alignment guide is used in locating principal points, in aligning photos for parallax measurements, and in measuring the length of the photo base or the separation of principal points.

stereoscopic study with either photo on top without the necessity of re-orientation. Finally, the alignment of the photos is checked by the graduated line (fig. 3), and the separation of the principal points is measured for use in the computations explained later in this paper.

Alignment of Wedge and Stereoscope

To view photos stereoscopically, the x-axis of the stereoscope must be parallel to the photo base. Parallax measurements must also be made parallel to the photo base. The wedge is placed over the photos with its converging lines approximately touching conjugate images and with the heavy tick marks aligned parallel to the photo base. The beginner, who has never viewed the parallax wedge or used height finders of the floating-dot type, may have difficulty in seeing the floating line of the wedge. This is almost always caused either by y-parallax[4/] or by incorrect separation of the photos. Since eyes differ, and since the beginner frequently does not know what to look for, a stereogram (fig. 4) will help accustom him to use of the wedge.

When viewed stereoscopically against the photographic model, a portion of the lines on the wedge appears to blend into a single sloping graduated line. This sloping line is used in reading heights. At the lower end, where the solid ground image causes the line to bend and separate, the graduation closest to ground level is read. Where the line appears to cut the more nebulous crown image, the graduation closest to the tip of the crown is read. The difference between these two readings is the parallax difference for the tree or stand. In actual practice, the ground level reading is taken as close as possible to the base of the tree. The wedge is then shifted towards the interpreter until the sloping line cuts the tree crowns at the point of measurement. So long as the proper tick marks are opposite, the wedge may be shifted at will, the measurement will be parallel to the photo base, and the difference in the two measurements will be directly related to the height of the tree or other object being measured.

4/ y-parallax: A lack of correspondence caused by a difference in perpendicular distances of two conjugate images from a vertical plane through the air base (1, p. 829).

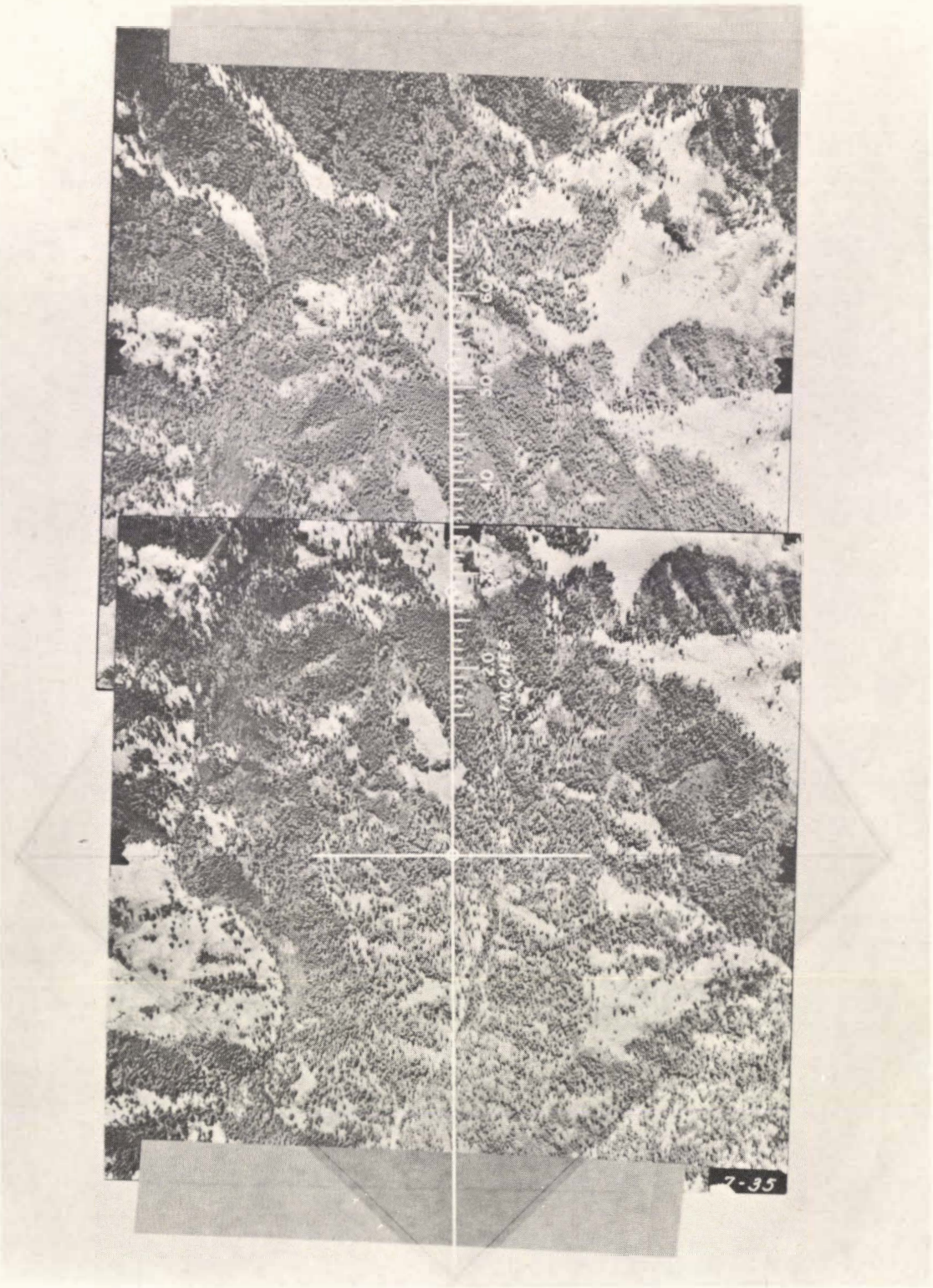

Figure 3.--Stereoscopic pair of 7"x7" aerial photos aligned for parallax measurement by the alignment guide. The guide is removed after the right-hand photo has been taped and before any measurements are made.

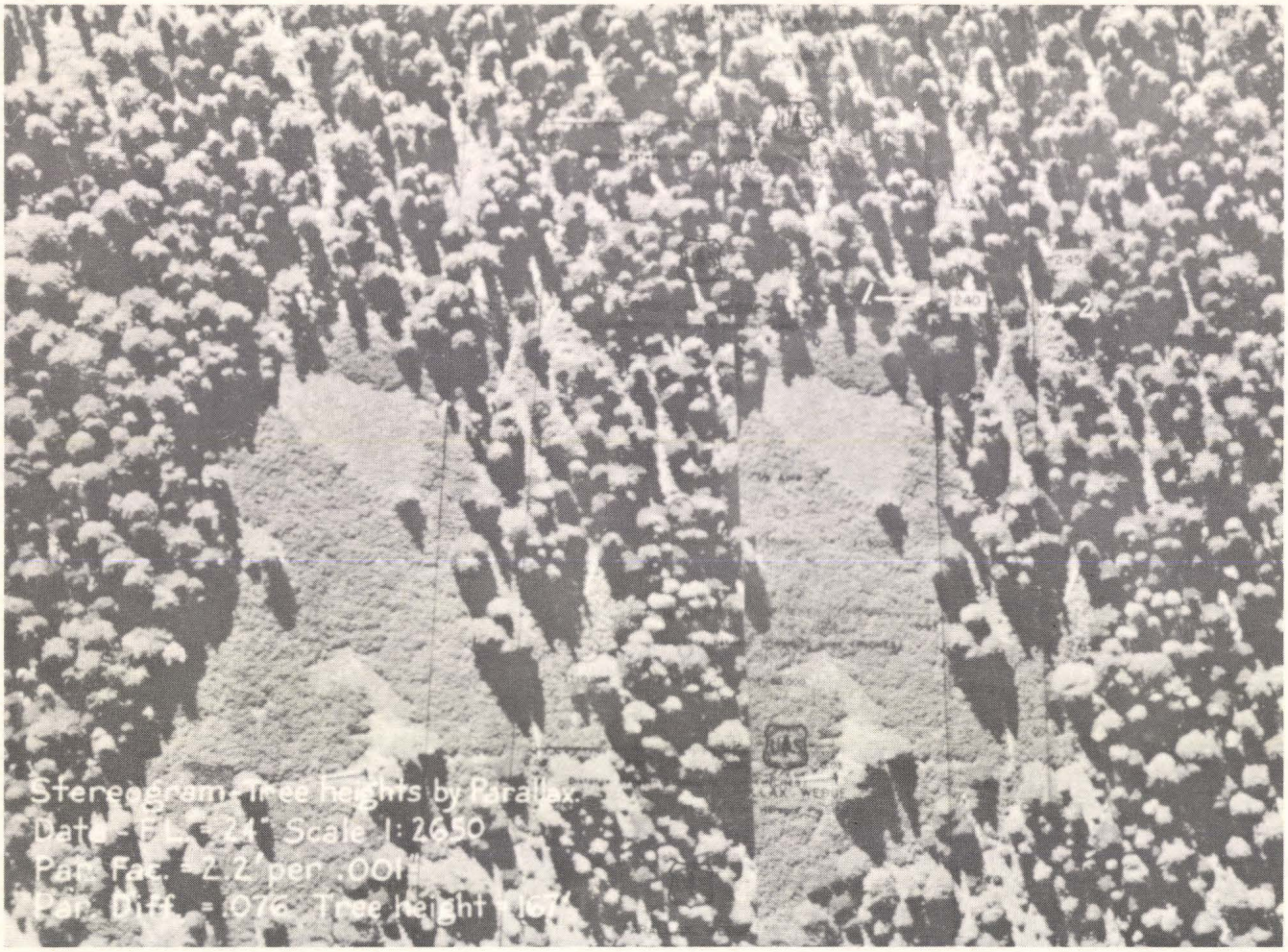

Figure 4.--The wedge viewed stereoscopically against the photographic model appears as a single, sloping, graduated line, which bends and separates at the ground level (2) and seems to cut through the more nebulous crown image (1). The parallax difference between (1) and (2) is directly related to the height of the tree.

MATHEMATICS OF PARALLAX MEASUREMENTS

The measurement of heights by parallax on truly vertical photos of level ground is merely a problem in triangulation (fig. 5). Instead of the base line, the photogrammetrist uses the air base, a line connecting the two camera stations fixed in space at the instant of exposure. It is represented on each contact print as a straight line connecting principal and conjugate principal points, called the photo base. The base angles are also fixed at the instant of exposure by the perspective of the aerial camera and are permanently recorded by the position of the photo images. The angles themselves are not measured, but rather the separation of conjugate images at the base of the tree and at the tip of its crown. The difference between these measurements is the parallax difference. It is related to the height of the tree and is indicated by the sum of the two lines a-b on prints 1 and 2 in figure 5.

The relationship between the height of the tree (ho) and the parallax difference (dp) measured on the photos depends on the height of the camera above the ground (H-h) and the parallax at the tree top (P_t). It is expressed in the formula:

$$ho = \frac{H-h \times dp}{P+dp} \qquad (1)$$

sometimes written

$$ho = \frac{H-h \times dp}{P_t}$$

where

H = height of camera above sea level
h = elevation of ground above sea level
dp = parallax difference for the tree
P = stereoscopic parallax at the ground level
P_t = parallax of tree top

In addition to (dp), the photo interpreter needs (P_t) measured on the photos and (H-h), which may be obtained from photo-map or photo-ground relationships.

Parallax at top of tree (P_t)

On vertical photos, all conjugate images in the same datum have the same parallax. Therefore it is customary on photos of level ground to substitute the easily measured photo base for (P), the parallax of the ground at the tree, and then add to it parallax difference (dp) to obtain (P_t).

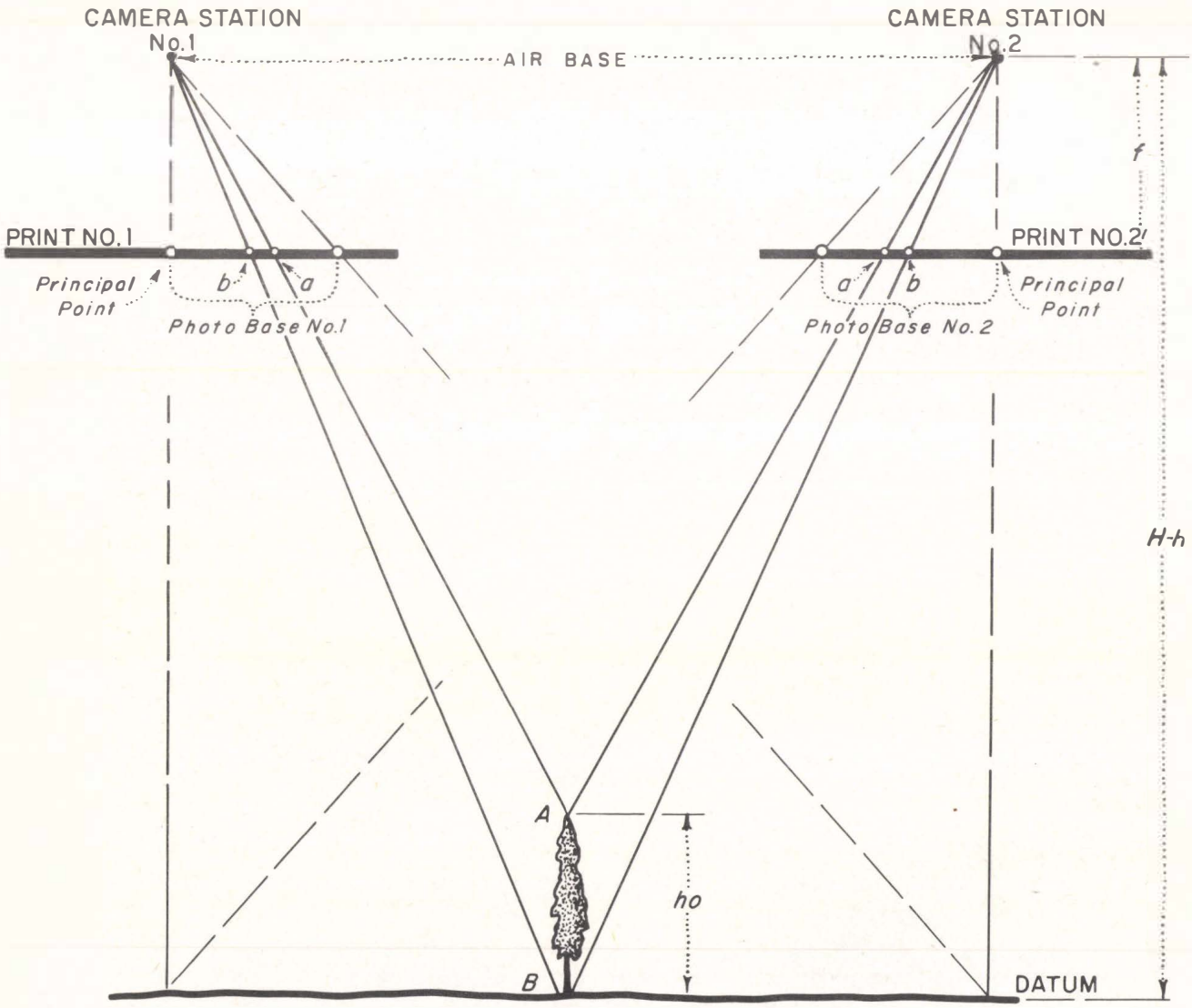

Figure 5.--Under ideal conditions, the measurement of tree heights consists of simple triangulation from an air base connecting the two camera stations.

However, this simplified procedure will result in errors if used on photos of mountainous terrain where each plot may lie at a different datum. Here (P_t) is determined by (1) measuring the separation between principal points on a pair of photos properly oriented for stereomeasurements, and (2) subtracting from this measurement the separation of the conjugate images for the tree top, as read on the parallax wedge. Parallax for any other point is obtained in like manner (1, p. 610).

For most purposes, the measurements may be rounded to the nearest 0.1 inch. For precise measurements on very large-scale photos, it is usually advantageous to make these measurements to the nearest 0.05 inch.

Elevation of Camera Above Ground (H-h)

Theoretical flying height above sea level (H) frequently can be obtained from contract specifications. From a topographic map, the elevation at the base of the tree (h) can be estimated, and from these data the approximate (H-h) may be computed. However, since the height above sea level (H) may vary within contract specifications, the flying height (H-h) obtained by this method should be used only in reconnaissance procedure.

A more precise computation uses the photo-map-scale relationship and a control line. Such a line should pass close to the principal point of one of the photos and be approximately bisected by it. It should terminate at points easily identified on both photo and map so that it can be measured precisely on each.

These measurements and other data are substituted in the following formula:

$$H-h = \left[\frac{fd^1}{d} + MSR + h_a\right] - h \qquad (2)$$

where
- f = focal length of camera in feet
- d^1 = map distance of control line in feet
- d = photo distance of control line in feet
- MSR = reciprocal of map scale (1/RF)
- h_a = average elevation of control line terminals in feet as read from topographic map

Although this problem can be solved at one time, the formula becomes less mysterious if visualized as follows:

(a) the term $\left[\frac{d^1\ MSR}{d}\right]$ that is $\frac{\text{map distance x map scale reciprocal}}{\text{photo distance}}$ gives us the PSR or photo scale reciprocal.

(b) when this is multiplied by focal length (f), the resulting value is flying height above the average elevation of the control line terminals.

(c) this flying height is then adjusted to height above sea level (H) by adding the average elevation at the control line (h_a) and is further converted to flying height above ground (H-h) by subtracting the elevation at the point of measurement (h).

If only planimetric maps are available, the flying height may be computed from the following formula:

$$H-h = \left[\frac{fdl\ MSR}{d}\right] \frac{P_a}{P_g} \qquad (3)$$

where P_a = average parallax of control line terminals in inches
P_g = parallax of the base of the tree in inches

By this formula, the flying height above the average elevation for the terminals of the control line is determined as before, and is then adjusted by the term $\left(\frac{P_a}{P_g}\right)$ to furnish the flying height above the ground point.

If no maps are available, the flying height above the ground can be computed by the following formula from a control line measured on the ground:

$$H-h = \left(\frac{fD}{d}\right) \frac{P_a}{P_g} \qquad (4)$$

where D = length of the control line on the ground in feet.

In this example, PSR is computed from the photo-ground relationship $\left(\frac{D}{d}\right)$ and multiplied by (f) to get flying height above the average elevation of the control line terminals. This flying height is adjusted as before by the term $\left(\frac{P_a}{P_g}\right)$ to height above the point of measurement.

Where numerous photos are being used, it may be expensive to get sufficient ground control. For such situations, a scheme of radial line triangulation discussed in the Manual of Photogrammetry (1, p. 409) can be used. The wing points and principal points can be plotted to a known scale and these points used for control.

PRACTICAL APPLICATION

In practice, particularly in the measurement of tree heights, the parallax problem may be solved for a constant (parallax factor) equal to a (dp) of 0.001 inch.[5/] This parallax factor is then multiplied by the (dp) of the tree to compute tree height in feet.

Example:

Values are substituted in the basic parallax formula (1) $ho = \frac{H-h \times dp}{P+dp}$

$H-h = 13{,}600$ feet
$P = 3.0$ inches
$dp = 0.001$ inch

$$\text{Parallax factor} = \frac{13{,}600 \times 0.001}{3.0 + 0.001} = 4.5318$$

$$\text{Parallax factor} = \frac{13{,}600 \times 0.001}{3.0} = 4.5333$$

This is rounded to 4.5 feet per 0.001 inch (dp) for use. Whether the parallax at the base of the tree (P) or the parallax at the tip of the crown (P_t), that is (P+dp), is used in computing the parallax factor has little importance in most tree measurements. However, when the (dp) term reaches an appreciable proportion of the (P), then this term can no longer be dropped safely from the formula in figuring tree height. This term (dp) becomes important when measuring tree height or elevation differences of 250 feet or more on 1:20,000-scale photos or trees of 100 feet or more on 1:5,000-scale photos.

The use of a constant in computing heights by parallax has resulted in two simplified techniques for determining this parallax factor; they are commonly known as the graphic (fig. 6) and tabular (tables 1, 2, and 3) techniques. Theoretically, these methods give about equally accurate results; they differ only in the ease and consistency with which they can be used. For this reason, use of the two techniques is shown below in parallel columns, and examples of their application are discussed simultaneously.

The following examples illustrate three specific but different situations that may be encountered by the interpreter when working with photos of mountainous terrain.

5/ Except when parallax measurements are to be made in mm. or 0.001 foot.

Table 1.--Parallax factors by parallax (P) and flying height (H-h)

Parallax (P) or (P+dp)	Flying height (H-h) in thousands of feet																																						Parallax (P) or (P+dp)
	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5	17.0	17.5	18.0	18.5	19.0	19.5	20.0	
Inches	Parallax factor (ho) in feet per 0.001 inch (dp)																																					Inches
1.5	1.3	1.7	2.0	2.3	2.7	3.0	3.3	3.7	4.0	4.3	4.7	5.0	5.3	5.6	6.0	6.4	6.7	7.0	7.3	7.7	8.0	8.3	8.7	9.0	9.3	9.7	10.0	10.3	10.7	11.0	11.3	11.6	12.0	12.4	12.7	13.0	13.3	1.5
1.6	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4.0	4.3	4.7	5.0	5.3	5.6	6.0	6.3	6.6	6.9	7.2	7.5	7.8	8.1	8.5	8.7	9.0	9.4	9.7	10.0	10.3	10.6	11.0	11.2	11.6	11.8	12.2	12.5	1.6
1.7	1.2	1.5	1.8	2.1	2.3	2.6	2.9	3.2	3.5	3.8	4.1	4.4	4.7	5.0	5.3	5.6	5.9	6.2	6.5	6.8	7.1	7.3	7.6	7.9	8.2	8.5	8.8	9.1	9.4	9.7	10.0	10.3	10.6	10.8	11.2	11.4	11.8	1.7
1.8	1.1	1.4	1.7	2.0	2.2	2.5	2.8	3.0	3.3	3.6	3.9	4.2	4.4	4.7	5.0	5.3	5.5	5.8	6.1	6.4	6.7	6.9	7.2	7.5	7.8	8.1	8.3	8.6	8.9	9.2	9.5	9.7	10.0	10.3	10.6	10.8	11.1	1.8
1.9	1.1	1.3	1.6	1.9	2.1	2.4	2.6	2.9	3.1	3.4	3.7	4.0	4.2	4.5	4.7	5.0	5.2	5.5	5.8	6.1	6.3	6.6	6.8	7.1	7.4	7.7	7.9	8.2	8.4	8.7	9.0	9.2	9.5	9.7	10.0	10.2	10.6	1.9
2.0	1.0	1.2	1.5	1.8	2.0	2.3	2.5	2.7	3.0	3.2	3.5	3.8	4.0	4.2	4.5	4.7	5.0	5.2	5.5	5.8	6.0	6.3	6.5	6.7	7.0	7.3	7.5	7.8	8.0	8.3	8.5	8.8	9.0	9.3	9.5	9.8	10.0	2.0
2.1	1.0	1.2	1.4	1.7	1.9	2.1	2.4	2.6	2.8	3.1	3.3	3.6	3.8	4.0	4.3	4.5	4.8	5.0	5.2	5.5	5.7	5.9	6.2	6.4	6.7	6.9	7.1	7.4	7.6	7.9	8.1	8.3	8.6	8.8	9.0	9.3	9.5	2.1
2.2	0.9	1.1	1.4	1.6	1.8	2.0	2.3	2.5	2.7	2.9	3.2	3.4	3.6	3.9	4.1	4.3	4.5	4.8	5.0	5.2	5.5	5.7	5.9	6.1	6.4	6.6	6.8	7.0	7.3	7.5	7.7	7.9	8.2	8.4	8.6	8.9	9.1	2.2
2.3	0.9	1.1	1.3	1.5	1.7	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.5	3.7	3.9	4.1	4.3	4.6	4.8	5.0	5.2	5.4	5.7	5.9	6.1	6.3	6.5	6.7	7.0	7.2	7.4	7.6	7.8	8.0	8.3	8.5	8.7	2.3
2.4	0.8	1.0	1.3	1.4	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7	3.9	4.1	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.7	6.9	7.1	7.3	7.5	7.7	7.9	8.1	8.3	2.4
2.5	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.8	7.0	7.2	7.4	7.6	7.8	8.0	2.5
2.6	0.8	1.0	1.2	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.3	6.5	6.7	6.9	7.1	7.3	7.5	7.7	2.6
2.7	0.7	0.9	1.1	1.3	1.5	1.7	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.3	3.5	3.7	3.9	4.1	4.3	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.7	5.9	6.1	6.3	6.5	6.7	6.9	7.0	7.3	7.4	2.7
2.8	0.7	0.9	1.1	1.3	1.4	1.6	1.8	2.0	2.1	2.3	2.5	2.7	2.9	3.0	3.2	3.4	3.6	3.7	3.9	4.1	4.3	4.5	4.6	4.8	5.0	5.2	5.4	5.5	5.7	5.9	6.1	6.3	6.4	6.6	6.8	7.0	7.2	2.8
2.9	0.7	0.9	1.0	1.2	1.4	1.6	1.7	1.9	2.1	2.2	2.4	2.6	2.8	2.9	3.1	3.3	3.4	3.6	3.8	4.0	4.1	4.3	4.5	4.6	4.8	5.0	5.2	5.3	5.5	5.7	5.9	6.0	6.2	6.4	6.6	6.7	6.9	2.9
3.0	0.7	0.8	1.0	1.2	1.3	1.5	1.7	1.8	2.0	2.2	2.3	2.5	2.7	2.8	3.0	3.2	3.3	3.5	3.7	3.8	4.0	4.2	4.3	4.5	4.7	4.9	5.0	5.2	5.3	5.5	5.7	5.8	6.0	6.2	6.3	6.5	6.7	3.0
3.1	0.6	0.8	1.0	1.1	1.3	1.5	1.6	1.8	1.9	2.1	2.2	2.4	2.6	2.7	2.9	3.1	3.2	3.4	3.5	3.7	3.9	4.0	4.2	4.3	4.5	4.7	4.8	5.0	5.2	5.3	5.5	5.6	5.8	6.0	6.1	6.3	6.5	3.1
3.2	0.6	0.8	0.9	1.1	1.2	1.4	1.6	1.7	1.8	2.0	2.2	2.3	2.5	2.7	2.8	3.0	3.1	3.3	3.4	3.6	3.7	3.9	4.1	4.2	4.4	4.5	4.7	4.8	5.0	5.1	5.3	5.4	5.6	5.8	5.9	6.1	6.3	3.2
3.3	0.6	0.8	0.9	1.1	1.2	1.4	1.5	1.7	1.8	2.0	2.1	2.3	2.4	2.6	2.7	2.9	3.0	3.2	3.3	3.5	3.6	3.8	3.9	4.1	4.3	4.4	4.5	4.7	4.8	5.0	5.2	5.3	5.5	5.6	5.7	5.9	6.1	3.3
3.4	0.6	0.7	0.9	1.0	1.2	1.3	1.5	1.6	1.7	1.9	2.1	2.2	2.3	2.5	2.6	2.8	2.9	3.1	3.2	3.4	3.5	3.7	3.8	4.0	4.1	4.3	4.4	4.6	4.7	4.8	5.0	5.1	5.3	5.4	5.6	5.7	5.9	3.4
3.5	0.6	0.7	0.9	1.0	1.1	1.3	1.4	1.6	1.7	1.9	2.0	2.1	2.3	2.4	2.6	2.7	2.8	3.0	3.1	3.3	3.4	3.6	3.7	3.9	4.0	4.1	4.3	4.4	4.6	4.7	4.9	5.0	5.1	5.3	5.4	5.6	5.7	3.5
3.6	0.6	0.7	0.8	1.0	1.1	1.3	1.4	1.5	1.6	1.8	2.0	2.1	2.2	2.4	2.5	2.6	2.8	2.9	3.0	3.2	3.3	3.5	3.6	3.8	3.9	4.0	4.2	4.3	4.4	4.6	4.7	4.9	5.0	5.1	5.3	5.4	5.6	3.6
3.7	0.5	0.7	0.8	0.9	1.1	1.2	1.4	1.5	1.6	1.8	1.9	2.0	2.2	2.3	2.4	2.6	2.7	2.8	3.0	3.1	3.2	3.4	3.5	3.7	3.8	3.9	4.0	4.2	4.3	4.5	4.6	4.7	4.9	5.0	5.1	5.3	5.4	3.7
3.8	0.5	0.7	0.8	0.9	1.1	1.2	1.3	1.4	1.5	1.7	1.8	2.0	2.1	2.2	2.4	2.5	2.6	2.8	2.9	3.0	3.2	3.3	3.4	3.6	3.7	3.8	3.9	4.1	4.2	4.3	4.5	4.6	4.7	4.9	5.0	5.1	5.2	3.8
3.9	0.5	0.6	0.8	0.9	1.0	1.2	1.3	1.4	1.5	1.7	1.8	1.9	2.1	2.2	2.3	2.4	2.6	2.7	2.8	2.9	3.1	3.2	3.3	3.5	3.6	3.7	3.8	4.0	4.1	4.2	4.4	4.5	4.6	4.7	4.9	5.0	5.1	3.9
4.0	0.5	0.6	0.8	0.9	1.0	1.1	1.3	1.4	1.5	1.6	1.7	1.9	2.0	2.1	2.3	2.4	2.5	2.6	2.7	2.9	3.0	3.1	3.2	3.4	3.5	3.6	3.7	3.9	4.0	4.1	4.2	4.4	4.5	4.6	4.7	4.9	5.0	4.0
4.1	0.5	0.6	0.7	0.9	1.0	1.1	1.2	1.3	1.4	1.6	1.7	1.8	2.0	2.1	2.2	2.3	2.4	2.5	2.7	2.8	2.9	3.0	3.2	3.3	3.4	3.5	3.7	3.8	3.9	4.0	4.1	4.3	4.4	4.5	4.6	4.8	4.9	4.1
4.2	0.5	0.6	0.7	0.8	1.0	1.1	1.2	1.3	1.4	1.5	1.7	1.8	1.9	2.0	2.1	2.3	2.4	2.5	2.6	2.7	2.9	3.0	3.1	3.2	3.3	3.5	3.6	3.7	3.8	3.9	4.0	4.2	4.3	4.4	4.5	4.6	4.8	4.2
4.3	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.3	1.3	1.5	1.6	1.7	1.9	2.0	2.1	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.4	3.5	3.6	3.7	3.8	3.9	4.1	4.2	4.3	4.4	4.5	4.7	4.3
4.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.3	2.4	2.5	2.6	2.7	2.8	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.4

Case 1. Reliable topographic maps are available

Although the theoretical contract flying height may be used,[6] the control line procedure is recommended. The bracketed term in formula 2 (p. 10 above) is the flying height above sea level (H).

Ground elevation at the tree is then estimated from the topographic map and subtracted from height above sea level to determine flying height above the tree to be measured (H-h).

Graphic	Tabular
On the graph (fig. 6), find the intersection of the vertical line above (H-h) with the sloping line (P_t), and project this to the left vertical axis to read parallax factor (feet per 0.001-inch dp)	In table 1, opposite the appropriate value (P+dp)[7] and below the flying height (H-h), read the parallax factor (feet per 0.001-inch dp)

Example:

H = 14,200 feet--computed from 10 control lines
h = 600 feet--estimated from the topographic map
H-h = 13,600 feet (the difference)
P_t or P+dp = 3.2 inches measured on photos
Parallax factor = 4.3 feet per 0.001 inch from graph or 4.2 feet per 0.001 inch from the table
dp = 0.018 inch--measured on the photo with parallax wedge
Tree height = 18 x 4.3 = 77.4 feet or 18 x 4.2 = 75.6 feet

The computed factor is actually 4.25 and the precise tree height, therefore, 76.5 feet. The small scale of the graph and the rounding of the table result in neither height being precise; but both computed heights are well within acceptable limits.

6/ It is usually advantageous to determine mean flying height for a given photo project by using about 10 control lines selected at random throughout the area. From these control lines, both the mean flying height and its standard error are computed. If this error is less than 5 percent, the mean flying height is acceptable.

7/ (P+dp) and (P_t) are synonymous, and both stand for the parallax at the top of the tree or other object measured.

Figure 6.--Tree height in feet for 0.001-inch difference in parallax, various flying heights, and various parallaxes.

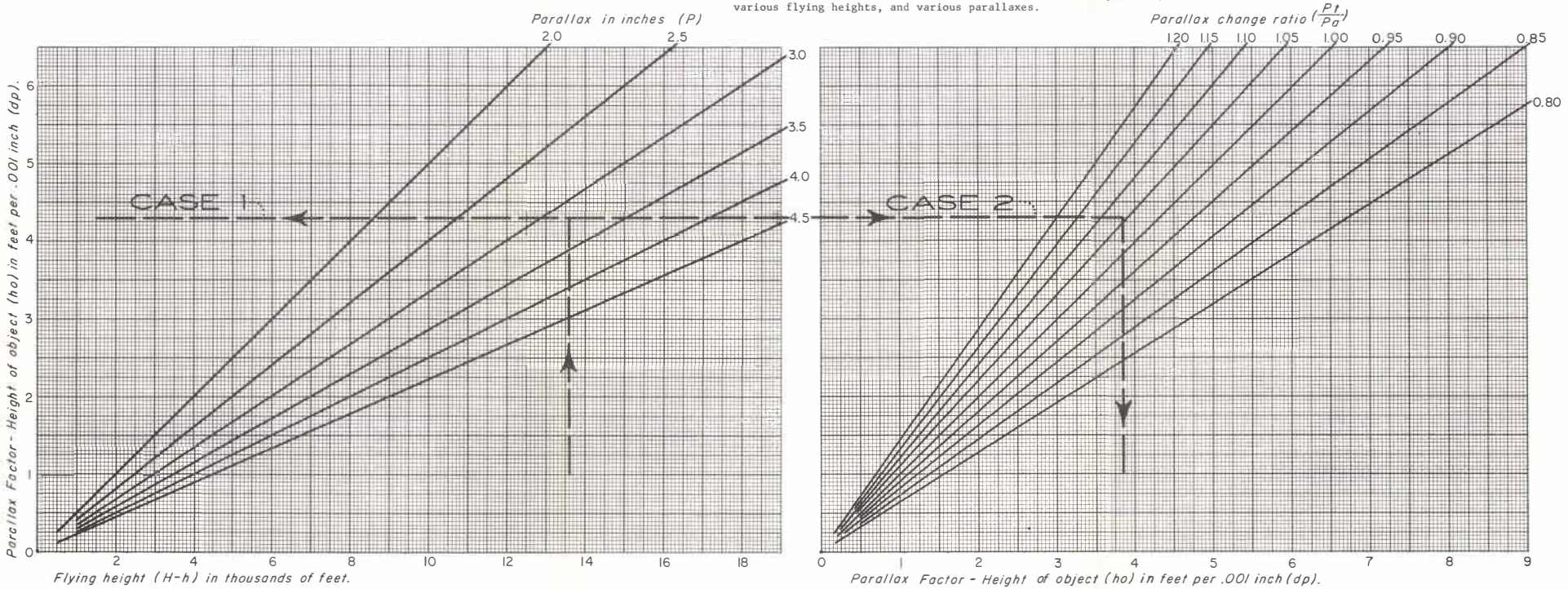
Parallax in inches (P)
2.0
2.5
3.0
3.5
4.0
4.5
CASE 1
CASE 2
Parallax change ratio (Pt/Pa)
1.20 1.15 1.10 1.05 1.00 0.95 0.90 0.85
0.80
Parallax Factor - Height of object (ho) in feet per .001 inch (dp).
0 1 2 3 4 5 6
2 4 6 8 10 12 14 16 18
Flying height (H-h) in thousands of feet.
0 1 2 3 4 5 6 7 8 9
Parallax Factor - Height of object (ho) in feet per .001 inch (dp).

Table 2.--Percent correction in parallax factor for difference in datum by parallax ratio

Parallax Ratio P_t/P_a	Adjustment	Parallax Ratio P_t/P_a	Adjustment	Parallax Ratio P_t/P_a	Adjustment	Parallax Ratio P_t/P_a	Adjustment
	Percent		Percent		Percent		Percent
0.80	155	0.85	140	0.90	125	0.95	111
0.81	152	0.86	137	0.91	122	0.96	109
0.82	149	0.87	134	0.92	119	0.97	106
0.83	146	0.88	131	0.93	117	0.98	104
0.84	143	0.89	128	0.94	114	0.99	102
1.00	100	1.05	91	1.10	83	1.15	77
1.01	98	1.06	89	1.11	82	1.16	75
1.02	96	1.07	88	1.12	80	1.17	74
1.03	94	1.08	86	1.13	79	1.18	73
1.04	92	1.09	85	1.14	78	1.19	71
1.20	70	1.25	65				
1.21	69	1.26	65				
1.22	68	1.27	64				
1.23	67	1.28	64				
1.24	66	1.29	63				

Table 3.--Photo scale reciprocals (PSR) by flying heights (H-h) in thousands of feet for 8.25-inch focal length camera

H-h	PSR	H-h	PSR	H-h	PSR	H-h	PSR	H-h	PSR
1.0	1,450	2.0	2,910	3.0	4,360	4.0	5,820	5.0	7,270
1.2	1,750	2.2	3,200	3.2	4,650	4.2	6,110	5.2	7,560
1.4	2,040	2.4	3,490	3.4	4,950	4.4	6,400	5.4	7,850
1.6	2,330	2.6	3,780	3.6	5,240	4.6	6,690	5.6	8,150
1.8	2,620	2.8	4,070	3.8	5,530	4.8	6,980	5.8	8,440
6.0	8,730	7.0	10,180	8.0	11,640	9.0	13,090	10.0	14,550
6.2	9,020	7.2	10,470	8.2	11,930	9.2	13,380	10.2	14,840
6.4	9,310	7.4	10,760	8.4	12,220	9.4	13,670	10.4	15,130
6.6	9,600	7.6	11,050	8.6	12,510	9.6	13,960	10.6	15,420
6.8	9,890	7.8	11,350	8.8	12,800	9.8	14,250	10.8	15,710
11.0	16,000	12.0	17,450	13.0	18,910	14.0	20,360	15.0	21,820
11.2	16,290	12.2	17,740	13.2	19,200	14.2	20,650	15.2	22,110
11.4	16,580	12.4	18,040	13.4	19,490	14.4	20,950	15.4	22,400
11.6	16,870	12.6	18,330	13.6	19,780	14.6	21,240	15.6	22,690
11.8	17,160	12.8	18,620	13.8	20,070	14.8	21,530	15.8	22,980
16.0	23,270	17.0	24,730	18.0	26,180	19.0	27,640	20.0	29,090
16.2	23,560	17.2	25,020	18.2	26,470	19.2	27,930	20.2	29,380
16.4	23,850	17.4	25,310	18.4	26,760	19.4	28,220	20.4	29,670
16.6	24,150	17.6	25,600	18.6	27,050	19.6	28,510	20.6	29,960
16.8	24,440	17.8	25,890	18.8	27,350	19.8	28,800	20.8	30,250

Case 2. Only planimetric maps are available

Flying height above the control line terminals is computed from the bracketed term in formula 3 (p. 11). The average parallax for the control line (P_a) and the parallax for the top of the tree (P_t) are also determined. A ratio $\left(\frac{P_t}{P_a}\right)$ is computed.

Graphic

On the graph (fig. 6), find the intersection of the vertical line above ($H-h_a$) with the parallax of the control line (P_a), and project this to the right until it intersects the sloping line of the parallax change ratio $\left(\frac{P_t}{P_a}\right)$. Project this intersection vertically to the bottom axis to read the adjusted parallax factor.

Tabular

In table 1, opposite the average value for the control line (P, or $P+dp$) and under the average flying height of the control line terminals ($H-h$) find the parallax factor for the control line. Then, in table 2, opposite the parallax change ratio $\left(\frac{P_t}{P_a}\right)$, read the parallax adjustment in percent. The product of these is the adjusted parallax factor.

Example:

$H-h_a$ = 13,600 feet (flying height at the control line)
P_a = 3.2 inches (average parallax of the control line)
P_t = 3.4 inches (parallax at the tree top)
$\frac{P_t}{P_a} = \frac{3.4}{3.2}$ = 1.063 (parallax-change ratio)

Parallax factor equals 3.85 feet per 0.001 inch from the graph, and 3.74 feet per 0.001 inch from the table.

dp = 0.018 inch measured on the photo with parallax wedge.
Tree height = 18 x 3.8 = 68.4 feet by the graphic method, and
18 x 3.7 = 66.6 feet by the tabular method.

Case 3. No maps available

Flying height is computed by means of formula 4 from a control line measured on the ground (p. 11). On project work, the radial line plot may be used in lieu of ground measurements of the control lines.

Both graphic and tabular techniques are the same as those described under Case 2.

This table will prove valuable to the interpreter using 8.25-inch focal length photography. It shows the relationship between H-h and PSR; that is:

$H-h$ = PSR x 0.6875 foot

Procedure

The step-by-step procedure that would be followed in making tree height measurements by either the graphic or tabular method is:

Given:

A stereo pair of aerial photos having unknown scale
A planimetric map with MSR = 24,000

Wanted:

Tree heights at a predetermined location

Procedure:

1. Orient the photos for parallax measurements

2. Lay out a control line on the photos and map, and measure it on each

   Photo distance (d) = 0.083 foot
   Map distance (d^1) = 0.048 foot

3. Scale (PSR) = $\frac{d^1 \text{ MSR}}{d} = \frac{0.048 \times 24{,}000}{0.083} = 13{,}879$

   Flying height ($H-h_a$) = $\frac{fd^1 \text{ MSR}}{d} = \frac{0.6875 \times 0.048 \times 24{,}000}{0.083} = 9{,}500$ ft.

   Table 3 may be used instead of this second formula to convert PSR to $H-h_a$.

4. The separation between principal points and between terminals of the control line is measured thus:

   Principal points = 5.50 inches
   Terminal 1 = 2.83 inches
   Terminal 2 = 2.85 inches

5. The average parallax for the control line (P_a) is computed by subtracting the average separation of control line terminals from the separation of the principal points.

   $P_a = 5.50 - \frac{2.83 + 2.85}{2} = 2.66$ inches

6. The parallax factor, at the control line, is read from table 1 or from figure 6, or may be computed by using flying height and parallax. From table 1, using parallax of 2.66 and $H-h_a$ of 9,500 the parallax factor = 3.6.

7. The separation of the conjugate images of the top and base of the tree is measured thus:

Top = 2.750 inches
Base = 2.766 inches
Parallax diff. (dp) = 0.016 inch

8. The parallax at the base and top of the tree is computed by subtracting the separation at each from the separation of the principal points.

$P_g = 5.500\text{-}2.766 = 2.734$
$P_t = 5.500\text{-}2.750 = 2.750$

9. A parallax ratio computed from $\left(\frac{P_t}{P_a}\right)$ is used to adjust the parallax factor for difference in elevation.[8] Ratio = $\frac{2.75}{2.66} = 1.034$. This adjustment may be made by tabular or graphic procedure. Table 2 indicates this adjustment should be 0.94. This is applied to the parallax factor of 3.6 to give an adjusted parallax factor for this location of 3.4 feet per 0.001 inch.

10. Tree height is the product of the parallax difference and parallax factor, or

ho = 3.4 x 16 = 54.4 feet

This height may be varied 1 or 2 feet by the way in which the tables are rounded and the number of places carried in the computations.

Some Tests of the Methods

To use the parallax wedge effectively, an interpreter simplifies and speeds up computations by certain assumptions and by using certain graphs or tables. However, graphs are seldom drawn on a large enough scale, and tables are rarely compiled in form voluminous enough to give answers as precise as those computed by formula. Therefore, some tests have been made that indicate the comparative precision of both of these working tools.

Measurements of 16 trees varying in height from 30 to 280 feet were used in each test. Tests were made at flying heights of 2,000 and 13,500 feet[9] and with parallaxes of 2 and 4 inches. They compare the answers

---

8/ Where many measurements are to be made on a single pair of photos the interpreter may determine the adjusted parallax factor for typical elevations throughout the stereopair.

9/ Photos made with 8.25-inch focal length camera would have a PSR of approximately 3,000 and 20,000.

obtained by formula with those obtained by graphs or tables. These tests lead to the following conclusions:

1. Both the graphic and tabular techniques are usable on trees of all heights when flying height exceeds 4,000 feet. At low flying heights over rough topography, parallax problems should always be solved by formula.

2. Adjustment for elevation differences when measuring tree heights results in better measurements. When elevations varied from -543 to +636 feet from mean datum, the standard error of estimate for trees of all heights was 10.6 feet without adjustment for elevation. This was reduced to 3 to 4 feet when computations were made by either the graphic or tabular method after factors were adjusted for elevation differences.

3. Differences between graphic or tabular expediencies and formula computations are much less on photos taken at the higher flying height. Reducing the flying height from 13,500 feet to 2,000 feet increased the standard error of estimate from 0.7 foot to 1.9 feet for trees less than 100 feet high.

4. Longer parallaxes[10/] tend to reduce the differences between graphs or tables and formula computations. Increasing parallax from 2 inches to 4 inches resulted in a standard error of estimate only one-half as large.

Relative Cost and Accuracy of Measurements

In spite of efforts to simplify procedures by use of graphs and tables, several simple yet time-consuming calculations remain which may be solved by slide rule. Where several height measurements at the same datum are needed, the parallax factor may be set on the slide rule, and tree height may then be read directly for various parallax differences. When adjustments for a change of datum are needed, the parallax ratios can be determined readily with the slide rule. The adjusted parallax factor is read and set for the new datum.

The cost of spot height measurements by any parallax instrument depends considerably upon the number of measurements that are made from each stereoscopic model; that is, the cost of setting up a pair of photos for parallax measurements is the same whether used for a single measurement or for a large number. The procedure described above for use of the photo alignment guide is very efficient. Repeated trials indicate that an experienced photo interpreter can set up photos and obtain a measurement of the distance between centers in approximately 1.5 minutes; about 0.8 minute of this time is required to locate principal and conjugate principal points on the photos. On photos of level country, on projects where (H-h) is known and

---

10/ When forward lap on 9"x9" photos is held at 60 percent, parallax (photo base) exceeds 3.5 inches. Increasing overlap to 75 percent reduces this to about 2 inches.

remains constant throughout the photo, the interpreter can find the proper factor in his parallax table (as table 1 above), set it on his slide rule, and complete the first parallax measurement in less than one additional minute. This means a total of 2.5 minutes from start of procedure. He can expect to set up photos and complete one measurement together with other interpretations per stereomodel at the rate of from 50 to 100 measurements per day. This number can be increased when several measurements are made from each setup.

Setup time is the same for stereomodels of mountainous country as for flat country, but lack of topographic maps, combined with variation in flying height, makes the determination of flying height at control datum and at point of measurement more time-consuming. Under these conditions, the time necessary to make parallax measurements of tree height may differ with each pair of photos.

The wedge and the procedure for tree measurement outlined in this paper offer considerable advantage in cost over the other measuring devices. For one thing, it is far simpler and faster to shift the wedge than to operate a micrometer wheel. The wedge, with its freedom from moving parts, seldom wears out; and it can be replaced for a few cents. The more complex instruments are extremely expensive and show little or no apparent advantage in accuracy of results.

Numerous studies have been designed to test the advantage of more expensive instruments over the simple parallax wedge (8). However, in the measurement of spot heights, no study has reported a significant advantage. This is to be expected since the wedge is calibrated in 0.002-inch (0.05-mm) calibrations; this is about the minimum unit of depth of perception for most individuals. Therefore, the limiting factor is the interpreter's depth perception; hence, the addition of finer calibrations found on more expensive instruments is not likely to increase accuracy.

SUMMARY

1. Results of the past 12 years' experience in measuring tree heights with the parallax wedge are assembled in this paper.

2. The orientation of photos, measurement of parallaxes, and conversion of parallax difference into height can be accomplished in as little as 2.5 minutes when flying height is known.

3. A prepared stereogram will assist the reader in becoming familiar with the parallax wedge by allowing him to view the wedge stereoscopically against the background of a properly aligned stereomodel.

4. Any parallax measurement can be visualized as a problem in triangulation with a base line defined by the two camera stations and angles represented by the parallax difference of the tip and base of the tree to be measured. Basic formulas may be solved by use of graphs or tables.

5. In photos of mountainous areas, parallax factors change as ground elevation changes. These adjustments are made conveniently with the aid of graphs or tables.

6. Limitations and errors in the use of the graphic and tabular techniques are shown and discussed.

7. Use of a slide rule can speed up computations of heights.

8. The cost of measuring heights by parallax is reduced by using the alignment guide and the parallax wedge procedure described.

LITERATURE CITED

(1) American Society of Photogrammetry.
1952. Manual of photogrammetry. 876 pp., Washington, D. C.

(2) Hutchison, O. Keith.
1948. Aerial photo alignment guide. Central States Forest Expt. Sta., Station Note 50.

(3) Moessner, Karl E.
1956. A parallax wedge for mountainous areas. Intermountain Forest & Range Expt. Sta., Research Note 39.

(4) _____
1955. The accuracy of stand height measurements on aerial photos in the Rocky Mountains. Intermountain Forest & Range Expt. Sta., Research Note 25.

(5) _____ Brunson, F. D., and Jensen, C. E.
1950. Accuracy of stand-height measurements on air photos. Central Forest Expt. Sta., Station Note 59.

(6) Rogers, Earl J.
1946. Use of the parallax wedge in measuring tree heights on vertical aerial photos. Northeastern Forest Expt. Sta., Forest Survey Note 1.

(7) Spurr, Stephen H.
1945. Parallax wedge measuring devices. Photogrammetric Engin. XI: 85-89.

(8) Worley, David P., and Landis, Glenn H.
1954. The accuracy of height measurements with parallax instruments on 1:12,000 photographs. Photogrammetric Engin. XX: 823-829.